Background:

- We expect the features across a neural network to be similar across tasks.
- Jacobian matrices are similar across tasks because Fisher information matrices are similar (Achille et al, 2019).

Description:

- Infinitely wide neural networks produce a kernel function that is often useful.
- What about the finite width regime?
- Use the Jacobian matrix so that the kernel becomes

$$K(x, x') := J_{\theta}(x)^T J_{\theta}(x')$$

- We share the parameters $\hat{\theta}$ across tasks.

- Work with the Jacobian matrix implicitly via matrix vector products

- Only need $J_{\theta}(x)v$ and $J_{\theta}(x)^Tv$ (Pearlmutter, 1994)
- Then use conjugate gradients and Lanczos as in **GPyTorch** (Gardner et al, 2018)

- To be able to mini batch computation, use fast Fisher-vector products

- Regression: $\mathbb{F}(\theta) = \frac{1}{N} J_{\theta}(x) J_{\theta}(x)^{T}$

- (Approximate) Fisher vector products:

$$\nabla_{\theta'} \mathrm{KL}(p(y|\theta)||p(y|\theta')|_{\theta'=\theta+\epsilon v} = \epsilon \mathbb{F}(\theta)v + \mathcal{O}(\epsilon^2||v||)$$

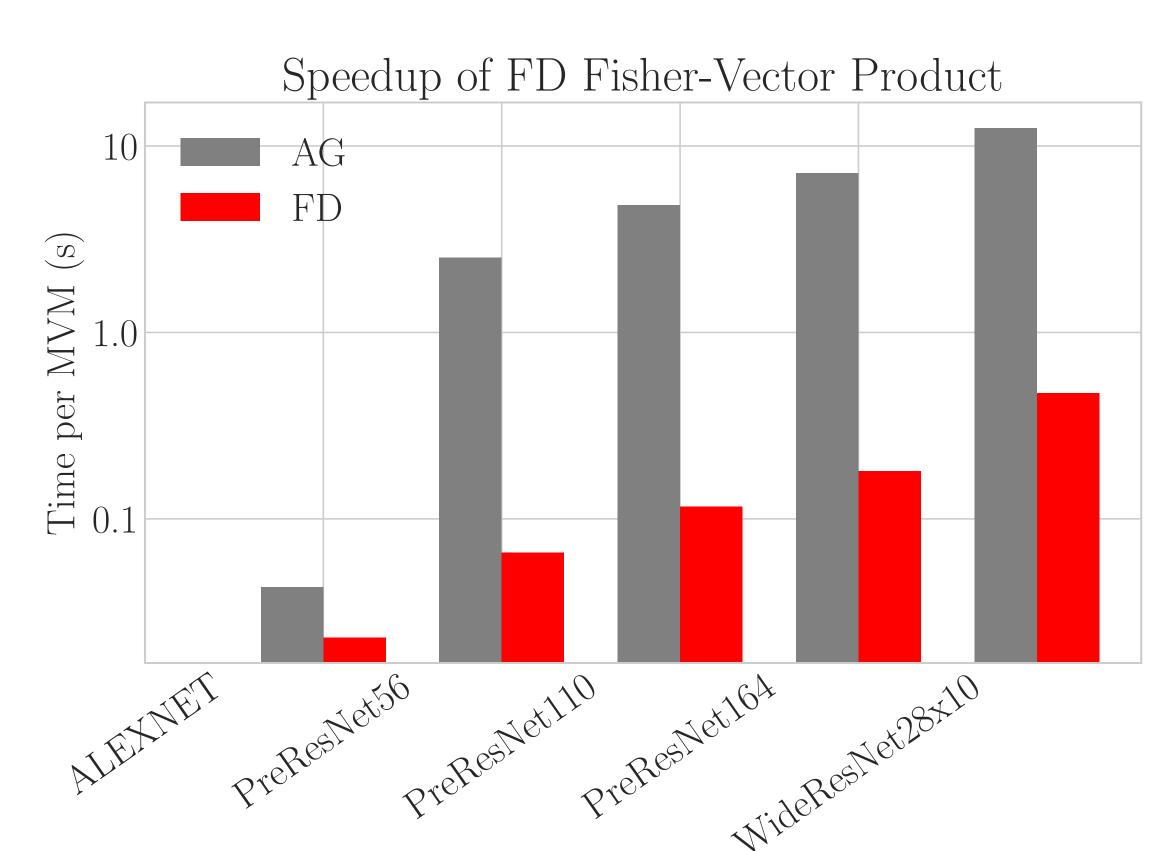
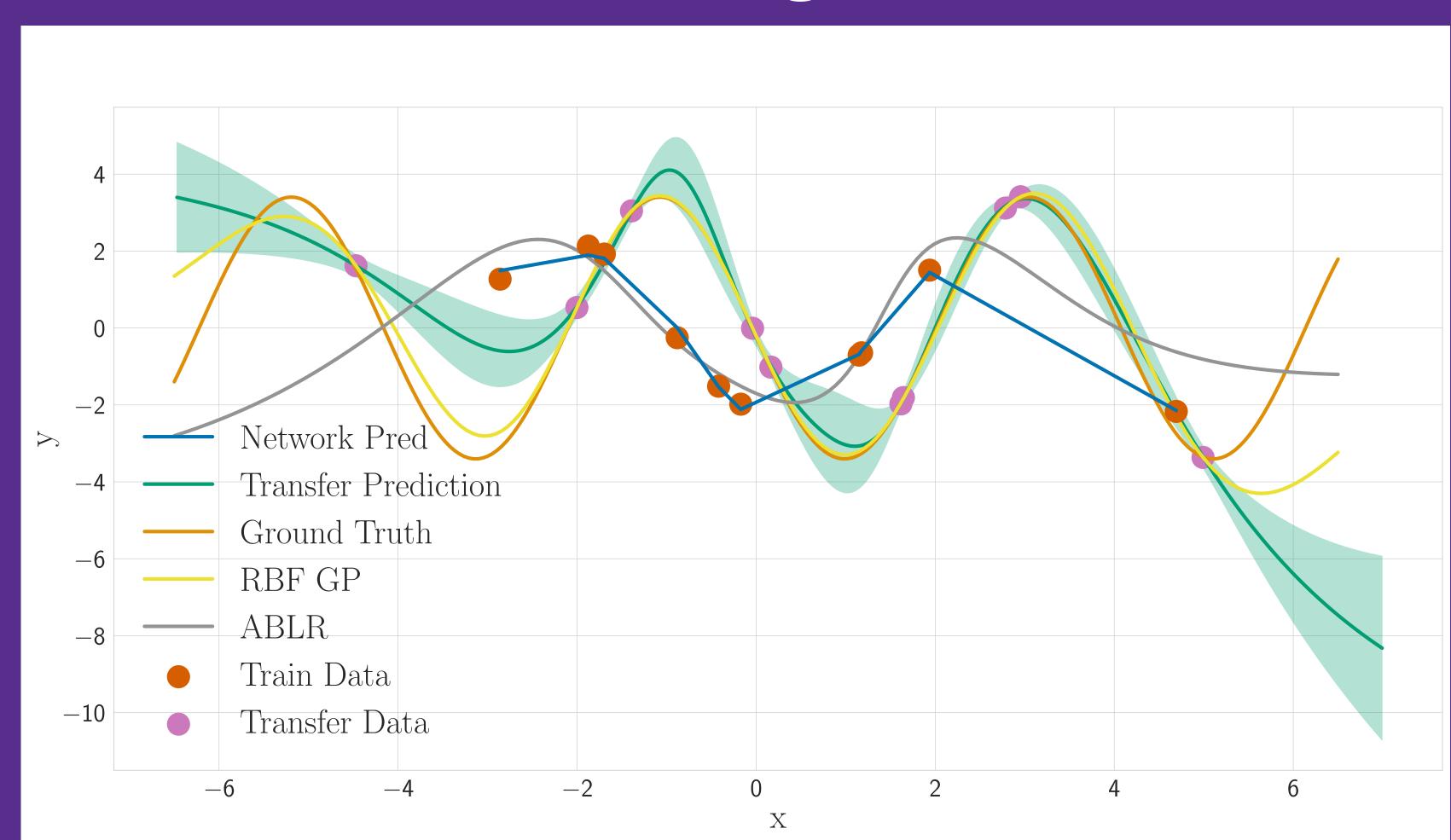


Figure: Speed comparison of Fisher vector products on CIFAR10. 30x speedup on most models.

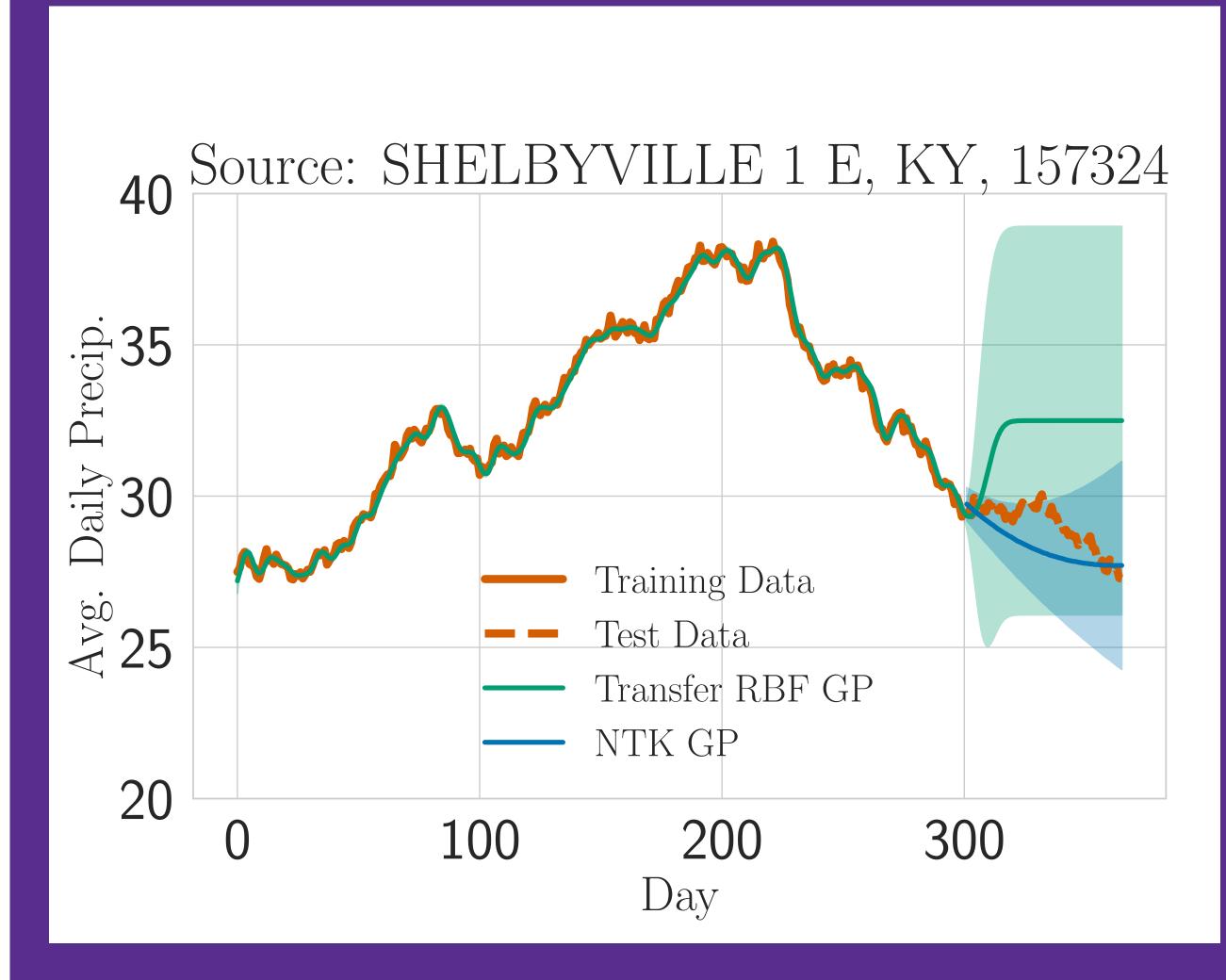
On Transfer Learning via Linearized Neural Networks

Linearizing trained neural networks produces a finite width neural tangent kernel that can be used for fast adaptation.

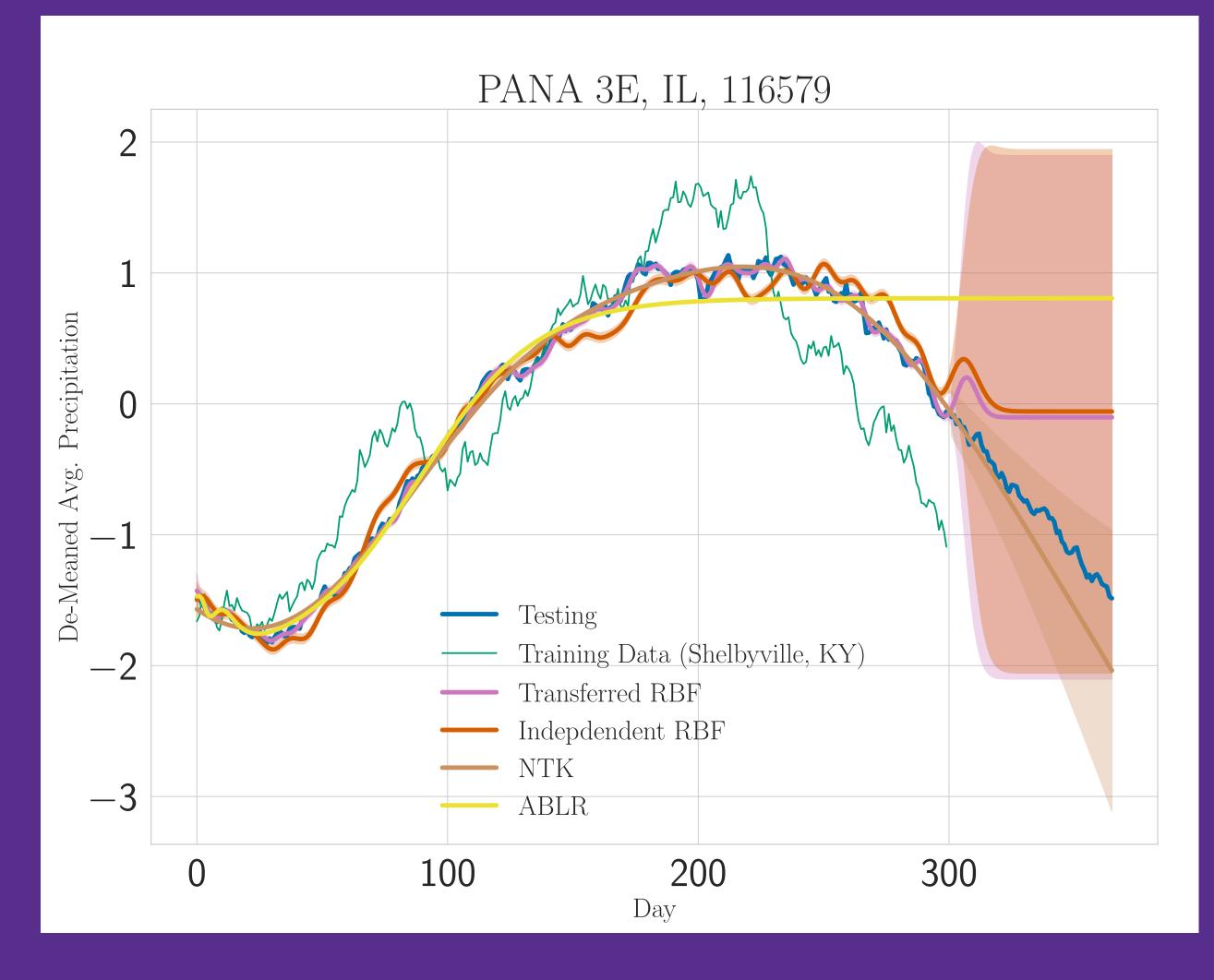
Transferring Sinusoids



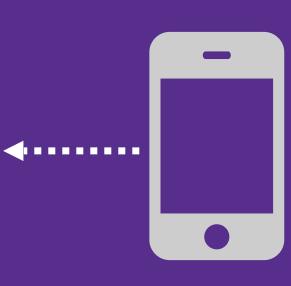
Source Task



Target Task







Take a picture to see the code and paper.

Probabilistic Model over Tasks

$$\theta_t' \sim p(\theta_t)$$

$$X_t)^T \theta_t + \mu_t$$

$$f_t = J_{\theta}(X_t)^T \theta_t + \mu_t$$
$$y_t | f_t \sim p(y_t | f_t)$$

- Evaluation: train on one task and then evaluate on every other task.

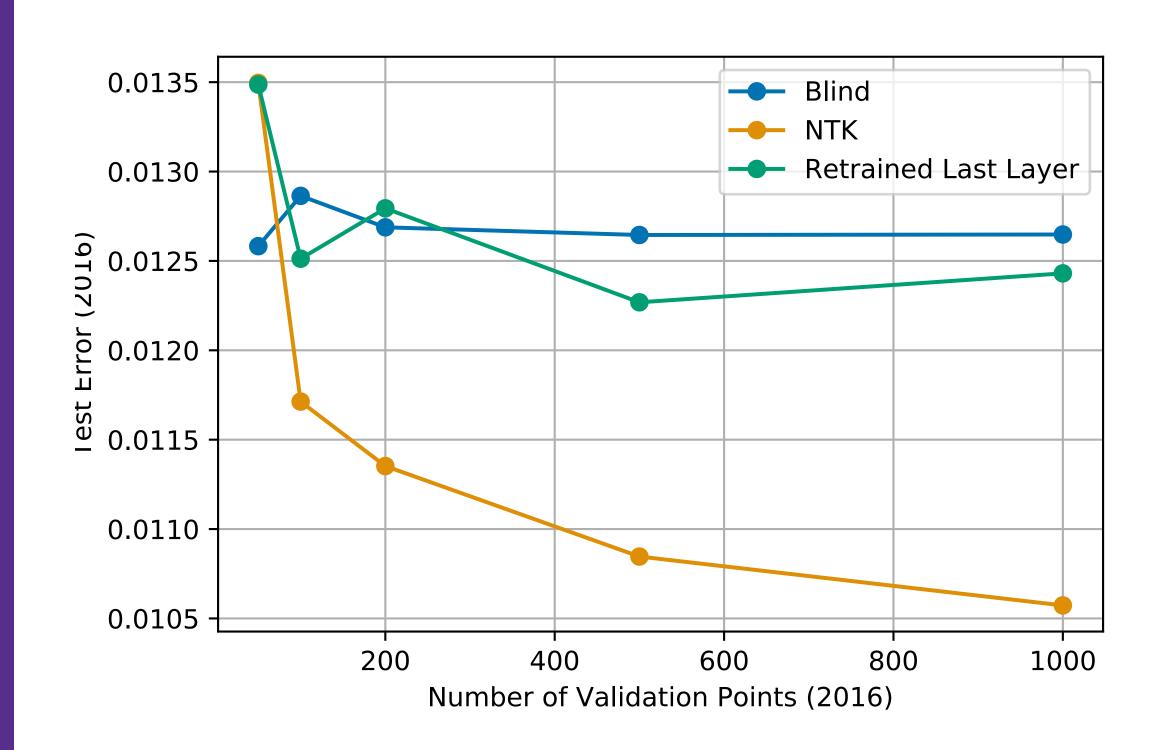


Figure: Large Scale malaria dataset from Malaria Global Atlas. Trained on 2000 data points from 2012, but tested on 2016 datagiven various amounts of validation data to the network. Finite NTK improves with more validation data from 2016 seen for a fixed testing set.

Future work will be to back propagate through the Jacobian vector products and convert into a closed form meta learning objective.

References:

- *Pearlmutter*, 1994. Fast exact multiplication by the Hessian, Neural Computation.
- Gardner et al, 2018. Gpytorch: Black Box Matrix Matrix Gaussian Process Inference with GPU Acceleration, NeurIPS.
- Achille et al, 2019. Task2vec: Task Embedding for meta-learning, arXiv:1902.03545.
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